Progressive Education Society's

Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16, NEP 2020 (Autonomous)

End Semester Examination: OCT / NOV 2024

***Total No. of Questions**: 5/31* ***Total No. of Pages**:3*

SECOND YEAR (S.Y.B.Sc. Blended)

MTH 301: Vector Calculus and Differential Equations.

(Semester III)

Program: B.Sc. Blended **Credits: 4**
Program Specific: B.Sc. Blended Chemistry **Time: 3 Hours**
Course Type: Open Elective **Max. Marks: 60**
Paper: IV **SET: A**

Instructions to the candidate:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Draw a well labelled diagram wherever necessary.*

SECTION: A

Q1) Answer the following **[1 X 10 =10]**

i) A matrix having only one column is called _____

a) Scalar matrix b) Diagonal matrix

c) Row matrix d) Column matrix

ii) If $u = (3,4)$ then $||u|| =$ _____.

a) 5 b) 3 c) 1 d) 4

iii) Two vectors u and v are said to be orthogonal if $< u, v > =$ _____.

a) 1 b) 2 c) 0 d) -1

iv) The eigen values of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are _____.

a) 1,1 b) 1,-1 c) 0,1 d) 0,-1

v) For the vector valued function $\bar{r}(t) = (t^2 - 3t)\bar{\imath} + (4t + 1)\bar{\jmath}$, $\bar{r}(0) =$ _____.

a) 0 b) $\bar{\jmath}$ c) $3\bar{\imath}$ d) $4\bar{\jmath}$

vi) $\int_0^1 x\, dx =$ _____.

a) 0 b) 1 c) 2 d) $\frac{1}{2}$

vii) If $y = x^2$ then $\frac{dy}{dx} =$ ___.

a) $2x$ b) 2 c) x d) 0

viii) If $\bar{F}(\bar{r}) = 2x\bar{\imath} - 2y\bar{\jmath}$ then $div\ \bar{F}$ = _____.

P.T.O.

a) 1 b) -1 c) 0 d) 2

ix) A function $f(x)$ is said to be an even function if $f(-x) =$ _____.

a) 0 b) $-f(x)$ c) 1 d) $f(x)$

x) The function $\sin x$ is a periodic function of period $L =$ _____.

a) 2π b) π c) 0 d) -2π

SECTION: B

Q2) Answer the following (Attempt any 5/7) **[5 X 2 = 10]**

i) State Cayley-Hamilton theorem

ii) Find determinant of the matrix $\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$.

iii) If $< u, v >= -3, ||u|| = \sqrt{19}, ||v|| = 3$ then find the angle between u and v.

iv) When a set of vectors is said to be orthonormal? Give an example.

v) State Green's Theorem'

vi) Compute the partial derivative of $\bar{F}(\bar{r}) = x^2\bar{\imath} + y^2\bar{\jmath}$ with respect to x and y.

vii) Find the gradient of $f(\bar{r}) = x^2 - y^2$.

SECTION: C

Q3) Answer the following (Attempt any 5/7) **[5 X 3 = 15]**

i) Find the characteristic equation and characteristic polynomial of A=$\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$.

ii) Show that for vectors u, v and w in an inner product space V,

<u-v, w> = <u, w> - <v, w>.

iii) Find the eigen values of A=$\begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}$.

iv) Compute the Partial derivative of the function $f(\bar{r}) = xy$ with respect to x & y.

v) Find the Eigen values of the matrix A=$\begin{bmatrix} -1 & 0 & -2 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Also find the eigen value of A^t.

vi) State and Prove Generalized Pythagoras Theorem.

vii) For the vector valued function given below

$$\bar{r}(t) = (t^2 - 3t)\bar{\imath} + (4t + 1)\bar{\jmath}$$

Evaluate $\bar{r}(0), \bar{r}(1)$.

SECTION: D

Q4) Answer the following (Attempt any 3/5) **[3 X 5 = 15]**

i) Verify that $u(x, y) = 2x(1 - y)$ is a harmonic function.

ii) Use Green's Theorem to evaluate $\oint_c xydx + x^2y^3dy$, where C is the triangle with vertices (0,0),(1,0),(1,2) with positive orientation.

iii) Find the $curl(\bar{F})$ for the vector fields $\bar{F} = (x + y)\bar{\imath} + (2xy)\bar{\jmath}$ on the plane.

iv) Find the flux of $\bar{F} = xy\bar{\imath} + yz\bar{\jmath} + xz\bar{k}$ outwards through the surface of the cube cut from the first octant by the planes x=1, y=1 and z=1.

v) Integrate $f(x, y, z) = x - 3y^2$+z over the line segment C joining the origin to the point (1,1,1).

SECTION: E

Q5) Answer the following (Attempt any 2/4) **[2 X 5 = 10]**

i) Find the Eigen values of the matrix A=$\begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$. Also find the eigen value of A^7.

ii) Determine whether the matrix A is diagonalizable. A=$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.

iii) Check whether <u, v>= $2u_1v_1 + 3u_2v_2$is an inner product on $\mathbb{R}^3$.

iv) State and prove Cauchy-Schwartz inequality.

*****