Progressive Education Society's

Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16, NEP 2020 (Autonomous)

End Semester Examination: Oct/Nov 2024

***Total No. of Questions**: 2/14* ***Total No. of Pages**: 02*

Second Year (Computer Science)

CMAT23201: Linear Algebra

(Semester III)

Program: BSc Comp05 **Credits: 2**
Program (Specific):B.Sc. Computer Science **Course Type: Minor**
Class: S.Y.B.Sc(Comp. Sci.) **Max. Marks: 30**
Course Code: CMAT23201

Instructions to the candidate:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Draw a well labelled diagram wherever necessary.*

Q.1) Answer the following. (Any 5 out of 7) [10 Marks]

i) Compute AB and A-B, where A = $\begin{bmatrix} -1 & 2 \\ 4 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$.

ii) Find the inverse of the matrix A= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

iii) Determine the determinant of the following matrix.

$$A=\begin{bmatrix} 1 & -2 & 0 \\ 1 & 3 & 1 \\ 5 & 0 & 2 \end{bmatrix}$$

iv) Is the set {(1, 1), (2, 3)} linearly independent? Justify.

v) Let V = R^3 , W= {(x, y, z) ∈ R^3 / 3x + 2y + z = 0}. Determine whether W is a subspace or not?

vi) Let T: $R^3 \rightarrow R^3$ be a linear transformation defined as T(x,y,z) =(x, y, 0). Find range T.

vii) Suppose T: $R^3 \rightarrow P^1$ is a linear transformation defined by T(a, b, c) = (-a + 2b +c) + (-b + c)x.

Determine which of the following vectors are in Ker(T).

A) u = (6, 2, 3) B) v = (-2, 1, -1).

Q.2) Answer the following. (Any 3 out of 5) [12 Marks]

i) Solve the following system using the Gaussian Elimination Method.

x + 2y + 3z =5

2x + 5y + 3z = 3

x + 8z = 17

P.T.O.

ii) Find the basis of row space of the following matrix A that consists of row vectors of A only

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 5 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 7 \end{bmatrix}$$

iii) Show that the vectors $v_1 = (1, 1, 1)$, $v_2 = (0, 1, 1)$, $v_3 = (0, 1, -1)$ are linearly independent in R^3.

iv) Find a standard matrix corresponding to the Linear Transformation T: $R^3 \rightarrow R^4$ defined as $T(x, y, z) = (x + y + z, -y + z, x - y, y + z)$.

v) Reduce the following matrix into row echelon form.

$$A = \begin{bmatrix} 2 & 10 & 8 & -26 \\ 3 & -1 & 2 & 5 \\ 2 & 2 & 3 & -4 \end{bmatrix}$$

Q.3) Answer the following. (Any 1 out of 2) [08 Marks]

i) Determine whether the matrix A is diagonalizable or not where $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.

ii) a) Find the inverse of the matrix using minors and co factors.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$$

b) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, determine whether $\det(A + B) = \det(A) + \det(B)$.