Progressive Education Society's

Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16 (Autonomous)

End Semester Examination: Oct. / Nov. 2024

***Total No. of Questions**: 5 / 17* *Total No. of Pages: 2*

F.Y.B.Sc.(Regular)

24MAT11101: Basic Course in Algebra

(Semester I)

Program: BscGen03 **Credits: 2**
Program Specific: Mathematics **Time: 2 Hours**
Course Type: Core Course **Max. Marks: 30**
SET: A

Instructions to the candidate:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Draw a well labelled diagram wherever necessary.***

Q1) Attempt **ANY FIVE** of the following: **[5 X 2 = 10]**

a) Define equivalence relation.
b) If X = {1 , 2 , 3}. Give the example on X which is symmetric and transitive.
c) Let the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 3x + 5$. Find f(t-1).
d) Find all primes which divide 40!.
e) State Remainder theorem.
f) Find the value of a if 1 is a root of the polynomial $f(x) = x^3 + ax - 2x + 7$.
g) State De Moiver's theorem.

Q2) Attempt **ANY THREE** of the following: **[4 X 3 = 12]**

a) Let $\mathbb{R}$ be the set of real numbers. For $x , y \in \mathbb{R}$ define the relation $\sim$ on $\mathbb{R}$ as $x \sim y$ if and only $|x| = |y|$. Show that $\sim$ is an equivalence relation on $\mathbb{R}$.
b) Let the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x – 4$ and $g(x) = \sin x$. Find i)fog ii)gof iii) fof iv) gog
c) Prepare the composition table for addition in $\mathbb{Z}_7$.
d) Define Congruence modulo n . For integers a , b , c , d if $a \equiv b \pmod n$ then prove that i)$ax \equiv bx \pmod n$ ii) $(a + x) \equiv (b + x) \pmod n$.
e) For $Z_1 , Z_2 \in \mathbb{C}$ prove that $| Z_1 + Z_2 | \leq | Z_1| + | Z_2 |$

Q3) Attempt **ANY TWO** of the following: **[2 X 4 = 08]**

a) Find the G.C.D. (71 , 50) and express in the form 71x + 50y. Find the values of x and y.
b) By using mathematical induction show that $2^n < 2n + 1$, for $n \geq 3$
c) If $f(x) = x^5 + 3x^4 + 5x^2 - 2$ and $g(x) = 2x^3 + 5x^2 - 7x - 4$. Find i) $f(x) + g(x)$ ii) $f(x) \cdot g(x)$
d) Express sin (3θ) and cos (3θ) in powers of sin θ and cos θ respectively.

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