Progressive Education Society's

Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16, NEP 2020 (Autonomous)

Total No. of Questions: 5

End Semester Examination: MAR / APR 2025

Total No. of Pages:3

FIRST YEAR (B.Sc. BLENDED)
24BLMT12401 : ALGEBRA
(Semester II)

Program: Science
Program Specific: B.Sc. Blended
Course Type: Elective

Credits: 4
Time: 3 Hours
Max. Marks: 60
SET: A

Instructions to the candidate:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of calculator is **NOT** allowed.*

Q1) Attempt the following. **[1 X 10 =10]**

a) Evaluate: $(-5+4i)\cdot(6-7i)$.
b) State any two types of matrix.
c) Define limit of a function.
d) If $\vec{a}=\vec{\imath}+2\vec{\jmath}-\vec{k}, \vec{b}=3\vec{\imath}-4\vec{k}$, then find $(\vec{a}\times\vec{b})$.
e) Define rank and nullity of linear transformation.
f) What is mean by differentiability of a function?
g) Write the standard basis of vector space for $V=\mathbb{R}^3$.
h) Define scalar triple product.
i) Perform elementary row operation R_2+2R_1 on the matrix $A=\begin{bmatrix}1 & -2\\ 5 & 6\end{bmatrix}$.
j) Find the distance between the points $P(7,-5,1)$ and $Q(-7,-2,-1)$.

Q2) Attempt ANY FIVE of the following. **[5 X 2 = 10]**

a) Find the determinant of the matrix $A=\begin{bmatrix}1 & 2 & 4\\ 7 & 2 & 0\\ 0 & 1 & 2\end{bmatrix}$.
b) Express $\frac{(9+i)}{(5-4i)}$ in the form of $x+iy$.
c) Write the non-homogeneous system of linear equations and represent it in the matrix form.
d) Evaluate the limit: $\lim\limits_{x\to 2}\frac{x-2}{x^2+x-6}$
e) Define linear transformation.
f) What is abstract vector space? Write its axioms.
g) What are the applications of Mean Value Theorem?

Q3) Attempt ANY FIVE of the following. **[5 X 3 = 15]**

a) Examine for consistency the following system of equations:

$$x + z = 2,$$
$$-2x + y + 3z = 3\,,$$
$$-3x + 2y + 7z = 4.$$

b) Prove that the function $f(x) = |x - a|$ is continuous at $x = a$ but not derivable at $x = a$.

c) Using $\epsilon - \delta$ definition of limit, prove that $\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$.

d) If A is an $m \times n$ matrix and $T: \mathbb{R}^n \to \mathbb{R}^m$ is multiplication by A, then prove that

i) nullity (T)=nullity (A)

ii) rank (T)=rank (A)

e) Reduce the matrix $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 3 & 1 & 0 \end{bmatrix}$ to the row-echelon form.

f) Prove that a square matrix A is invertible if and only if A is non-singular.

g) Find the equation of the plane passing through the points $P_1(1, 2, -1)$, $P_2(2, 3,1)$ and $P_3(3, -1,2)$.

Q4) Attempt ANY THREE of the following. **[3 X 5 = 15]**

a) If $\vec{A} = \vec{\imath} - 2\vec{\jmath} - 3\vec{k}$, $\vec{B} = 2\vec{\imath} + \vec{\jmath} - \vec{k}$, and $\vec{C} = \vec{\imath} + 3\vec{\jmath} - 2\vec{k}$, then find i) $\vec{A} \cdot (\vec{B} \times \vec{C})$ ii) $(\vec{A} \times \vec{B}) \cdot \vec{C}$

b) If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ 1 & 2 \end{bmatrix}$, verify that adj $(AB) = (adj\ B) \cdot (adj\ A)$.

c) Discuss the continuity of the function $f(x)$ where,

$$f(x) = \frac{x^2}{a} - a, \quad \text{for } 0 < x < a$$
$$= 0, \quad \text{for } x = a$$
$$= a - \frac{a^3}{x^2}, \quad \text{for } x > a$$

d) Solve the equation : $x^2 - i = 0$.

e) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $T(1,0,0) = (0,0), T(0,1,0) = (1,1)$ and $T(0,0,1) = (1, -1)$. Compute $T(4, -1,1)$ and determine the rank and nullity of T.

Q5) Attempt ANY TWO of the following. **[2 X 5 = 10]**

a) State and prove De Moivre's Theorem.

b) Define inverse of a matrix. If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, then find A^{-1} (if it exists) by adjoint method.

c) State and prove Lagrange's Mean Value Theorem.

d) Test the following equation for consistency and solve it, if consistent.

$$2x - y - 5z + 4w = 1,$$
$$x + 3y + z - 5w = 18\,,$$
$$3x - 2y - 8z + 7w = -1.$$

*****